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DEPARTMENT ELECTRICAL ELECTRONICS
COURSE ENG 281 [ASSIGNMENT 11]
MATRIC NO 16/ENG04/018

1. The parametric equations of a curve are as given in equation (1) and (2)

$$x = \cos t + t\sin t \quad --- (1)$$

$$y = \sin t - t\cos t \quad --- (2)$$

In terms of t, determine

(i) an expression for the radius of curvature (R) and

(ii) expressions for the coordinates (h, k) of the centre of curvature

Soln

i) Give (taking) R as radius of curvature

$$R = \frac{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$$

We take $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$; $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

$\frac{dy}{dt} = \cos t - (-t\sin t + \cos t)$ (from eqn 2 $y = \sin t - t\cos t$)

$\frac{dy}{dt} = \cos t + t\sin t - \cos t = \underline{t\sin t}$ --- (3)

from eqn (1) $x = \cos t + t\sin t$

$\frac{dx}{dt} = -\sin t + (t \times \cos t + 1 \times \sin t)$

$\frac{dx}{dt} = -\sin t + t\cos t + \sin t = t\cos t$ --- (4)

but $\frac{dt}{dx} = \frac{1}{t\cos t}$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = t\sin t \times \frac{1}{t\cos t} = \frac{\sin t}{\cos t}$ (recall $\frac{\sin\theta}{\cos\theta} = \tan\theta$)

$\frac{dy}{dx} = \underline{\tan t}$

for $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$ but no derivative for x

$\therefore \frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{dt}{dx}$ (from eqn 4 $\frac{dt}{dx} = \frac{1}{t\cos t}$

$\therefore \frac{d}{dt}\left(\tan t\right) \times \frac{1}{t\cos t} \Rightarrow \sec^2 t \times \frac{1}{t\cos t}$

(because $\frac{dy}{dx} \tan t = \sec^2 \theta$) $\left(\sec^2\theta = \frac{1}{\cos^2\theta}\right)$

$\frac{d^2y}{dx^2} = \frac{\sec^2 t}{t\cos t}$

$\therefore \frac{d^2y}{dx^2} = \frac{1}{\cos^2 t} \div t\cos t = \frac{1}{\cos^2 t} \times \frac{1}{t\cos t} = \frac{1}{t\cos^3 t}$

then $R = \frac{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = \frac{\left(1+\tan^2 t\right)^{\frac{3}{2}}}{\frac{1}{t\cos^3 t}}$

recall $1 + \tan^2\theta = \sec^2\theta \Rightarrow \frac{(\sec^2 t)^{\frac{3}{2}}}{\frac{1}{t\cos^3 t}} = \frac{\left(\sqrt{\sec^2 t}\right)^3}{\frac{1}{t\cos^3 t}}$

$R = \frac{\sec^3 t}{\frac{1}{t\cos^3 t}} \Rightarrow \sec^3 t \times t\cos^3\theta = \frac{1}{\cancel{\cos^3 t}} \times \frac{t\,\cancel{\cos^3 t}}{1}$

$\therefore R = t$

but $\tan\theta = \frac{dy}{dx}$ $\therefore \tan\theta = \tan t \therefore \theta = t$

(angle of slope)

ii a) $h = x_1 - R\sin t$

from eqn 1 $x = \cos t + t\sin t$ $R_1 = t$

$h = \cos t + \cancel{t\sin t} - \cancel{t\sin t}$

$h = \cos t$

(b $K = y_1 + R\cos t$ (from eqn 2 $y_1 = \sin t - t\cos t$

$K = \sin t - \cancel{t\cos t} + \cancel{t\cos t}$

$K = \sin t$